A comparative study of three Monte Carlo codes for β-detector simulations

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In order to determine the vector coupling constant and to test the unitarity of the Cabibbo-Kobayashi-Maskawa (CKM) matrix, one has to make precise measurements of nuclear masses, β-branching ratios and half-lives [1]. The measurements of half-lives and branching ratios are performed in a simple, but very precise counting station at our institute. A typical ”on-line” branching ratio experiment (see, for example Ref. [2, 3]) involves collection of the accelerator-produced radioactive nuclei on the tape of a tape-transport system that rapidly moves the collected sample to a position located between a scintillaton detector and a well-calibrated 70% HPGe γ-detector. Coincident β-γ events are collected and recorded. In order to completely understand all systematic effects contributing to the branching ratio measurements, one must determine the relative efficiency of the scintillator as a function of β-particle energy because the various γ-ray peaks follow β-transitions with different end-point energies and their observed relative intensities are affected by the small differences in β -detection efficiency. The work reported here continues an investigation, previously reported [4], of the response function of β-particles from standard open β-sources (*eg.* ^{207}Bi). Here we are concerned with the question of which Monte Carlo (MC) code is more suitable for simulations of low-energy electron transport. We present a comparison of Monte Carlo simulations with three general purpose codes: Geant4 (version 4.9.0), Penelope and EGSnrc.

The completely realistic geometric model for Geant4 [5], which was chosen for MC simulation, is shown in Figure 1 of Ref. [4]. The β-detector consists of a 1-mm thick BC404 scintillator material coupled via a plastic optical pad made from lucite to a photomultiplier. The scintillator-optical-pad assembly is enclosed in an opaque cylindrical shield made from 1.5-mm-thick PVC. The opening in the scintillator end of the PVC shield is covered with a pin-hole-free, 5-μm-thick havar foil. The β-particles enter the detector assembly through this foil with essentially negligible energy loss.

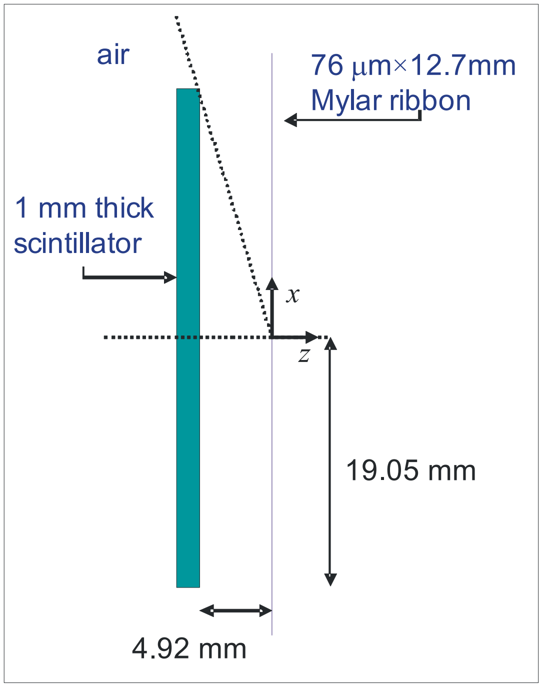

Figure 1. Simplified geometry used for MC simulations of a scintillating β-detector efficiency with three general purpose codes: Geant4, EGSnrc and Penelope.

The Geant4 Simulation Toolkit includes a series of packages for the simulation of the electromagnetic (EM) interactions [6], specialized for different particle types, energy range or approach in physics modeling. In this work, we considered only positrons and used the low-energy and standard EM physics models in Geant4. The Penelope package is an alternative low-energy implementation in Geant4; it is a re-engineered version [7] of the original stand alone Penelope MC code [8, 9]. EGSnrc is a general purpose package designed for the MC simulation of the coupled transport of electrons and photons in arbitrary geometries [10]. EGSnrc utilizes versions of Moliere theory that describe the global effect of many small-angle scattering interactions. The multiple scattering (MSC) model used in Geant4 belongs to the class of condensed simulations that use model functions to determine the angular and spatial distributions after a step. The stand-alone Penelope code inherently differs in how it handles MSC by additionally including hard collisions (such as large-angle Mott scattering). As this may be significant for a proper description of back-scattering of low-energy betas, it is important to compare the results of the different codes in our specific geometry and energy range of interest.

In order to compare the codes, a simple MC experiment was performed. First, all materials in the simulated geometry were replaced by vacuum, except for the plastic detector. As a source, we took a point-like mono-energetic positron source that emitted isotropically. Under these conditions, the total efficiency of the detector was approximately the same as its geometrical efficiency, which could be calculated from the fractional solid angle subtended by the scintillator relative to the source. The total (geometrical) efficiency was calculated by the formula

$$\varepsilon = \frac{\Omega}{4\pi} = \frac{1}{2}\left(1 - \frac{H}{\sqrt{R^2 + H^2}}\right),$$

where H is the distance between the detector and source, R is the radius of the scintillator, and Ω is the solid angle of the cone. See Figure 1. In our case, this leads to an efficiency of 37.5 %. Figure 2(a) shows the simulated results – obtained with the Geant4 low-energy EM package – for mono-energetic β^+-particles with energies in the range of 200 keV to 20 MeV. All MC-simulated results show very good agreement with the prediction based on the geometrical model. In the MC calculations we included all β-

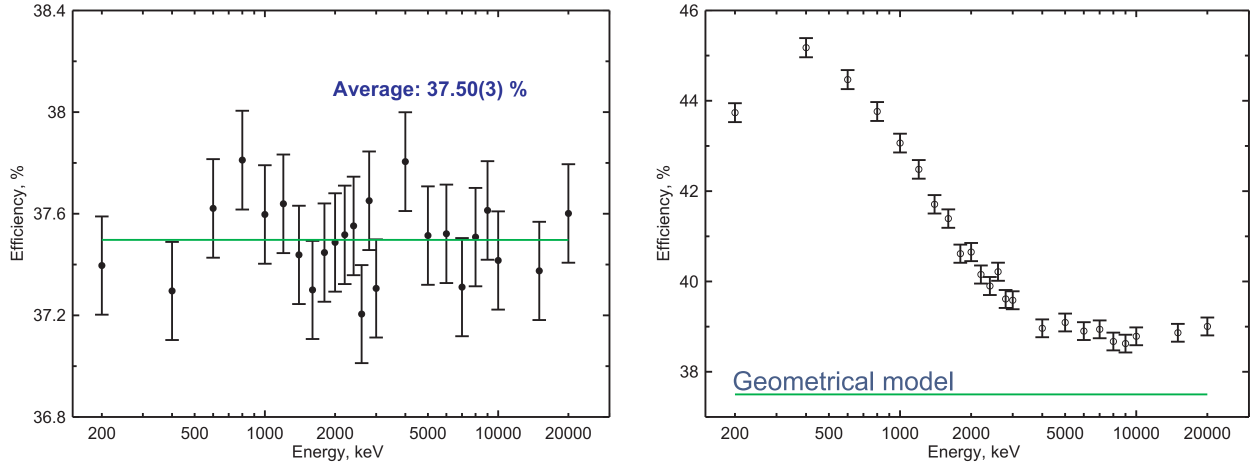

Figure 2. (a) Efficiencies calculated for a point-like β-source at 4.92 mm from the front of the scintillator in vacuum. Results are shown for monoenergetic β-particles emitted at various energies in the range from 200 keV to 20 MeV. (b) Efficiencies calculated for the same scintillator but with the point-like β-source placed in the center of a 76 μm thick aluminized mylar tape, also in vacuum.

particles that lost non-zero energy in the scintillator (*i.e.* no "cut-off" energy was used). Naturally, the introduction of a "cut-off" energy would shift the absolute efficiency to a lower value.

To our surprise, when the same point-like source was placed into the center of a 76 μm-thick aluminized mylar tape – the condition that applies to our actual on-line β-decay measurements –the total efficiency of the β-detector changes drastically throughout the whole energy range, but especially for β-particles energies below 4 MeV. See Figure 2(b). Although the mylar tape is very thin, it causes enough (back)scattering to increase the total efficiency of the detector by up to 20%. Even for betas at energies above 4 MeV, although the total efficiency remains more or less constant, it is still 4% higher than the result obtained by the simple geometrical estimation.

Figure 3(a) shows a comparison of the results from the Penelope code with the results from three different Geant4 EM physics models: low-energy EM model, standard EM model and a model that claims to emulate the Penelope code. (The EGSnrc results are not shown since they were virtually indistinguishable from the Penelope results.) Although we expected the results from the stand-alone Penelope to be the same as those from the Geant4 Penelope EM model, that was not what we found. This apparently is a consequence of the fact that the reengineered Penelope EM model contained in Geant4 uses a different method to describe the multiple scattering of beta particles than did the original Penelope code.

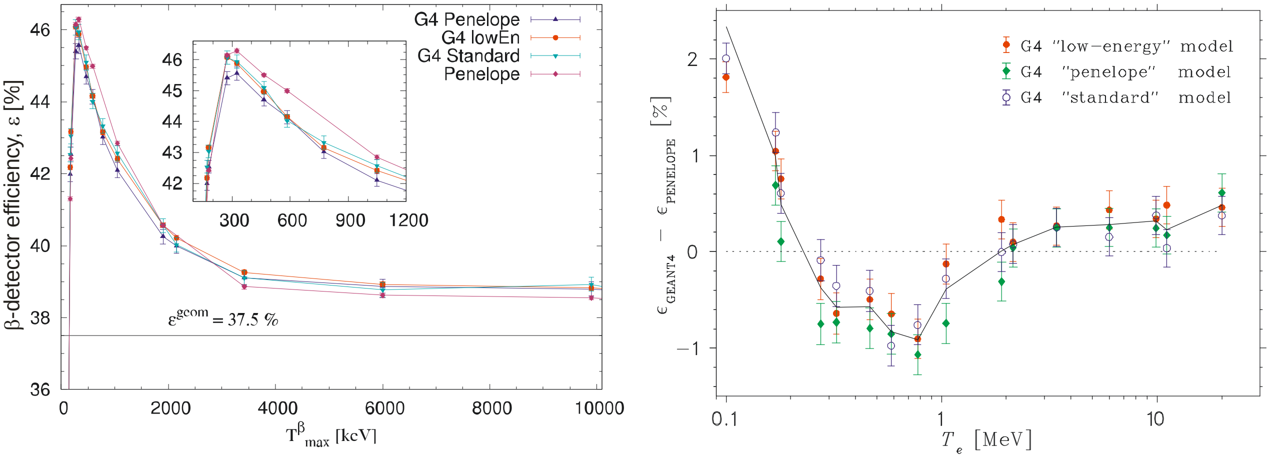

Figure 3. (a): β-detector efficiency of the plastic scintillator placed in air at a distance of 4.92 mm from a point-like source of mono-energetic positrons located at the center of aluminized mylar tape with thickness 76 µm; four different MC calculations are shown. The horizontal line is the geometrical efficiency. (b): The difference in efficiencies between Penelope and Geant4 with various EM physics models. The horizontal dotted line represents the stand-alone Penelope MC results.

Figure 3(b) shows the difference between the results for the efficiency calculations performed with the stand-alone Penelope program versus the three different EM physics models available in Geant4. Above 300 keV the difference in the efficiencies between the various Geant4 EM models and Penelope is less than 1%, with even better agreement above 1 MeV. The biggest discrepancies are in the energy region from 100 keV up to 1 MeV, although even there it is never larger than 2%.

The differences in MC codes that we have observed in these thin materials with very simplified geometry appear not to be entirely negligible. However, the resulting absolute efficiencies for monoenergetic positrons from point-like sources placed in the center of the aluminized mylar tape do not exceed 2% for the energies below 1 MeV and are well below 1% for energies between 1 MeV and 12 MeV. A comparison of Monte Carlo simulations for realistic sources – ^{207}Bi, ^{22}Na and ^{60}Co – is described in a separate report [11].

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